

Student Name: _____ **Solution** _____ Student ID: _____

Given the operation times as follow:

Instruction memory access time = 190 ps,
 Register file read access time = 150 ps,
 ALU delay (basic instructions) = 190 ps,

Data memory access time = 190 ps
 Register file write access = 150 ps
 ALU delay (multiply/divide) = 550

Assume the following instruction mix:

30% ALU, 15% multiply/divide, 15% load, 15% store, 15% branch, and 10% jump.

- What is the total delay for each instruction class and the clock cycle for a **single cycle** MIPS implementation?
- Assume we fix the clock cycle to 200 ps for a **multi-cycle** CPU, what is the average CPI? And speedup over a single cycle?
- If we convert the multi-cycle into 5 stages **pipelined** CPU, what would be the execution time and speedup over the single cycle and the multi-cycle?
- If the load instructions cause one stall to be inserted into the pipeline, and the multiply/divide instructions cause two stalls to be inserted, what would be the increase in the pipeline CPI over the ideal CPI? Re-calculate speedups of part (c)?

Solution:

Instruction Class	Instruction Memory	Register Read	ALU	Data Memory	Register Write	Total Delay (ps)
Basic ALU	190	150	190		150	680
Mul & Div	190	150	550		150	1040
Load	190	150	190	190	150	870
Store	190	150	190	190		720
Branch	190	150	190			530
Jump	190	150				340

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- a. Max delay is when performing mult/div, thus for single cycle design, the clock cycle time is equal to **1040 ps**, i.e. clock rate = 0.96 MHz
- b. For multi-cycle design with a clock period of 200 ps, i.e. clock rate = 5 MHz:

Basic ALU = 4 stages → 4 cycles

Multiply & Divide = 4 stages but ALU requires three cycles → 6 cycles

Load = 5 cycles

Store = 4 cycles

Branch = 3 cycles

Jump = 2 cycles

$$\text{Average CPI} = 0.3 * 4 + 0.15 * 6 + 0.15 * 5 + 0.15 * 4 + 0.15 * 3 + 0.1 * 2 = 4.1$$

Speedup of multi-cycle over single-cycle;

$$\text{Sup}(s \rightarrow m) = (\text{multi clk rate}/\text{single clk rate}) * (\text{single CPI}/\text{multi CPI})$$

or

$$S(s \rightarrow m) = ((\text{single cycle execution time}) * s - \text{CPI}) / ((\text{multi clk cycle time}) * m - \text{CPI})$$

$$= (1040 * 1) / (200 * 4.1) = 1.27$$

In case you wanted to calculate average execution time;

Average CPU execution time = Cycle time * CPI * NumberOfInstructions

$$T = C * \text{CPI} * I$$

For any instruction, its execution time = 200ps * 4.1 = 820 ps

- c. Ideal pipeline CPI = 1 running at 200 ps;

Execution time for any instruction = 200ps * 5 = 1000 ps

However Mult&Div requires three ALU cycles which means the pipeline should really be a seven stages pipeline, thus its execution time is equal to 1400 ps. The following speed up calculations hold true whether the pipe is 5 or 7 stages. This is because the pipeline CPI is independent of the number of stages.

Speedup pipeline over single cycle;

$$S(s \rightarrow p) = (T_s * s \text{CPI}) / (T_p * p \text{CPI}) = (1040 * 1) / (200 * 1) = 5.2$$

Speedup pipeline over multi cycle;

$$S(m \rightarrow p) = (T_m * m \text{CPI}) / (T_p * p \text{CPI}) = (200 * 4.1) / (200 * 1) = 4.1$$

d. None ideal pipeline;

$$\begin{aligned} pCPI'' &= 1 + \text{pipeline cycles with stalls} = 1 + (\text{load stalls} + \text{div\&multiply stalls}) \\ &= 1 + (0.15 \times 1 + 0.15 \times 2) = \mathbf{1.45} \end{aligned}$$

Speedup pipeline over single cycle;

$$S(s \rightarrow p)^* = (T_s * s - CPI) / (T_p * p - CPI'') = (1040 * 1) / (200 * 1.45) = \mathbf{3.586}$$

Speedup pipeline over multi cycle;

$$S(m \rightarrow p) = (T_m * m - CPI) / (T_p * p - CPI'') = (200 * 4.1) / (200 * 1.45) = \mathbf{2.83}$$